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ANALYSIS OF THE DISPLACEMENTS
OF THE GROUND SURFACE DUE
TO A MOVING VEHICLE

by

Robert L. Schiffman

The Winslow Laboratories
Rensselaer Polytechnic Institute
Troy, N.Y.

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Robert L Schiffman
Associate Professor of Soil Mechanics
Rensselaer Polytechnic Institute

SYNOPSIS

The problem of the displacement of the ground surface under the action of a moving vehicle is treated from an analytical point of view.

The physical mechanisms of soil displacements are presented for the purpose of establishing various models of behavior of the soil system Visco-elastic models of soil behavior are postulated on the basis of laboratory tests and field observations.

A theoretical study is made of the displacements of a loaded plate moving across the soil at relatively slow speeds. This study is based on theories of visco-elasticity.

The results of the study are presented in terms of analytical expressions and curves.

A physical interpretation of the results is presented, relating vehicular and soil parameters.

INTRODUCTION

The proper design of a vehicle requires knowledge and consideration of all the components affecting vehicle mobility.

Of all the components which play a part in vehicle mobility, the soil is the most variable and least understood. The soil also exhibits the most complex properties of all the mobility components.

For reason of soil behavior complexity, the mobility design of a vehicle must not only be related to the soil, but to the vehicle's use on a particular class of soils.

The use characteristic, as far as the soil is concerned, is the weight of the vehicle and the speed of its movement.

Soil can be grouped into three basic types, these being granular, clay, and mixtures of granular and clay-soils. Each of these soil groups has certain significant unifying characteristics. These characteristics can be used to predict soil-vehicle behavior for a given vehicle mobility consideration.

This paper develops certain soil deformation principles in terms of the vehicle characteristics of speed and weight.

The soil considered is a clay. The vehicle characteristics are treated as a loaded plate subjected to repetitive loading.

In general, the behavior of the soil will be restricted
to isotropic materials that follow linear laws with small strain.
The stress-strain-time relations will be for a homogeneous con-
tinuum, although considerations of soil as a multi-phased
system will be inherent in the analysis.

BEHAVIOR LAWS

Elastic Behavior

The stress-strain behavior for a homogeneous linear, isotropic, elastic material, exhibiting small strains is as follows:

$$\sigma_{ij} = \frac{3K-2G}{3} \epsilon_{kk}\delta_{ij} + 2G\epsilon_{ij} \tag{1a}$$

$$\delta = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{1b}$$

Where:

K = Bulk Modulus

G = Shear Modulus

Equation (1) can be separated into volume change components (σ_{ij}) and (ϵ_{ij}), and deviator components (s_{ij}) and (e_{ij}).

$$\sigma_{ij} = 3K\epsilon_{ij} \tag{2a}$$

$$s_{ij} = 2Ge_{ij} \tag{2b}$$

The relations between the volumetric and deviator stress and strain components are:

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} \tag{3a}$$

$$e_{ij} = \epsilon_{ij} - \frac{1}{3}\epsilon_{kk}\delta_{ij} \tag{3b}$$

The displacement of a loaded plate on the surface of an elastic half-space, as shown in Figure I, can be expressed as follows:

$$u_z = \alpha \, \frac{3K + 4G}{4G(3K + G)} \, \frac{P}{\sqrt{A}}$$ (4)

Where:

w = Surface displacement of plate.

α = Geometric constant depending on shape of plate and position where displacement is measured.

P = Total load on plate.

A = Area of loaded plate.

Thus, the surface displacement of a plate can be expressed in terms of the elastic properties and the imposed load.

Visco-Elastic Behavior

A visco-elastic stress-strain equation differs from the elastic relation only in the fact that the stresses and strains are related by time derivatives. The general linear visco-elastic stress-strain equation is:

$$\sum_{n=0}^{n} A_n \, \frac{\partial^n \sigma_{ij}}{\partial t^n} = \sum_{m=0}^{m} B_m \frac{\partial^m \epsilon_{ij}}{\partial t^m}$$ (5)

Equation (5) can be written in operator form in terms of volume change and deviatoric components [1,2].*

$$R(\sigma_{ij}) = S(\epsilon_{ij}) \qquad (6a)$$

$$M(s_{ij}) = N(e_{ij}) \qquad (6b)$$

The operators, R, S, M, N, are time derivative operators multiplied by the material constants associated with the equivalent stresses and strains. By this definition, the bulk and shear modulus are first order operators. As a result, the visco-elastic stress-strain law is:

$$\sigma_{ij} = \frac{MS-NR}{3RM} \epsilon_{kk}\delta_{ij} + \frac{N}{M} \epsilon_{ij} \qquad (7)$$

Since the visco-elastic operators are linear, the elastic surface displacement equation is readily converted to a visco-elastic equation.

$$N(2MS + NR)w = M(MS + 2NR)P(t) \qquad (8a)$$

$$w = \frac{u_z\sqrt{A}}{\alpha} \qquad (8b)$$

Thus, given the visco-elastic properties of a soil system and the type of loading, the time-displacement relation can be developed.

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* Numbers in brackets refer to the Bibliography at the end of the paper.

CLAY-SOIL BEHAVIOR

The behavior of clays can be separated into three components.

1. Volume change behavior

2. Deviator behavior

3. Relation between volume change and deviator behavior.

The first two components listed above are known, at least for a series of general conditions. The third component is as yet quantitatively and qualitatively unknown. For the purposes of this paper, the coupling of volume change and deviator is relatively unimportant and will not be considered. It must be understood, however, that in a prototype design, the interdependence of volume change and deviator properties will substantially alter the magnitude of these properties, and must be considered.

Volume Change

The fundamental volume change theory of clay-soils is the Terzaghi theory of Consolidation [3].

This theory considers the clay-soil mass to be a simplified saturated structure of incompressible particles and incompressible fluid.

From a rheologic point of view, the Terzaghi Theory develops volume change as a Kelvin material [4].

The Terzaghi theory was developed for long term

behavior under conditions of sustained loading. For a con-
dition of short repetitive loads, the theory must be augmented
[5]. The augmented theory of volume change behavior is rheo-
logically a Burgers material [6], under stress (σ_o) and strain
(ϵ_o).

The Burgers model, as shown in Figure 2 is in itself
not sufficient.

The bulk modulus (K_2) cannot be a constant. If it
were constant, the terminal volumetric deformation would be
independent of the imposed stress. From tests and analyses
by Bridgman [7], Seed [8], and Murnaghan [9], a tentative
bulk modulus law is proposed.

$$K_2 = K_o + \lambda \sigma_o \qquad (9)$$

Where

K_o = Residual bulk modulus

λ = Stress increment modulus

Under sustained loading, the proposed volume change
model must be compatible with the observations of terminal
deformation. Thus, the volumetric viscosity (ζ_1) must
increase with a decreasing volume, approaching infinity in
the limit. As the fluid is squeezed out, the pores tend
to close and the internal forces opposing fluid motion will
increase. As a result, the viscosity (ζ_1) will increase
both as a function of time and pressure. As a first ap-
proximation, the volumetric viscosity (ζ_1) can be considered

to follow an exponential law.

$$\zeta_1 = \zeta_0 \exp(a\sigma_0 t) \qquad (10)$$

Where:

ζ_0 = Initial viscosity

a = Constant

The bulk modulus (K_1) expresses the effects of initial and final deformation and instantaneous reversibility. As such, it is dependent on the magnitude of the imposed stress and the time of loading to the point of unloading. Since the effects of imposed stress magnitude were considered in equation (9), the variation of (K_1) can be considered only in terms of deformation reversibility. Observations of repeated loading tests [10], have indicated that the instantaneous reversibility decreases with number of repetitions and the time of previous sustained loading. Since the problem considers only a limited number of repetitions, the modulus (K_1) will be considered to vary only in terms of the time of loading.

$$K_1 = \mu_0 + \mu_1(1 - e^{-bt}) \qquad (11)$$

Where:

$\mu_0 \mu_1$ = Bulk moduli

b = Constant

The second volumetric viscosity (ζ_2) can be thought of as the initial permeability of the soil and as such, can

be considered to remain constant.

The total volume change relation can be expressed as follows:

$$\epsilon_0 = \frac{\sigma_0}{K_1} + \int_0^t \frac{\sigma_0}{\zeta_1} dt + C \exp\left[-\int \frac{dt}{T_2}\right]$$

$$+ \exp\left[-\int \frac{dt}{T_2}\right] \int_0^t \frac{\sigma_0}{\zeta_2} \exp\left[\int \frac{dt}{T_2}\right] dt \qquad (12a)$$

$$T_2 = \frac{\zeta_2}{K_2} \qquad (12b)$$

Where:

ϵ_0 = Volumetric strain

The response (ϵ_0) due to a hydrostatic compression (σ_0) is:

$$\epsilon_0 = \frac{\sigma_0}{\mu_0 + \mu_1(1-e^{-bt})} + \frac{1}{a\zeta_0}(1-e^{-a\sigma_0 t}) + \frac{\sigma_0}{K_0 + \lambda\sigma_0}(1-e^{-t/T_2}) \qquad (13a)$$

$$T = \frac{\zeta_0}{K_0 + \lambda\sigma_0} \qquad (13b)$$

For a single load-unload cycle, the response is sketched in Figure 3.

As shown in Figure 3, the strain time behavior has all the observed phenomena for volume change of clay-soils. The effect of individual parameters in terms of such soil entities as moisture content, type of clay mineral, degree of saturation, temperature, etc., must as yet be determined. These however, will only change the magnitude of the properties.

Deviator Behavior:

The deviator behavior of clay-soils in terms of rheologic properties has been described by Geuze, [10, 11, 12, 15], Tan [10, 11, 12, 13, 14], and Josselin de Jong [15], and Haefli [16].

These results based on creep tests report the average shear stress (τ) as a function of the time rate of shear strain (D). The reported experiments show that for low stress levels below a yield value (f_o) the deviator behavior is elastic. Between (f_o) and (f_1) the behavior is viscous, and for stress levels above (f_1) the behavior exhibits shear thinning until the shear strength (s) is reached. The magnitudes of (f_o), (f_1), (s) and the slopes of the curve are dependent on the soil properties such as moisture content, previous load history, clay mineral, degree of saturation, temperature, etc. There is as yet insufficient experimental evidence to relate these magnitudes.

The discontinuity of the behavior can be treated analytically by considering the clay to be a thinning

thixotropic Burgers material under stress (τ) and strain (γ). The deviator viscosity behaves as follows:

$$\frac{1}{\eta_1} = \phi_0 e^{c\tau} + \phi_2 e^{-dt} \tag{14}$$

The Burgers model is shown in Figure 2.

For the Burgers deviator model, the flow relation is:

$$D = \left[\frac{1}{\eta_1} + \frac{1}{\eta_2}\right]\tau + \frac{1}{G_1}\frac{\partial \tau}{\partial t} - \frac{1}{\eta_2 T_2} e^{-t/T_2} \int_0^t \tau e^{t/T_2}\, dt \tag{15a}$$

$$T_2 = \frac{\eta_2}{G_2} \tag{15b}$$

Examining equations (15) with (η_1) as defined in equation (14), and considering time-strain equilibrium:

$$D = \phi_0 \tau\, e^{c\tau} \tag{16}$$

The relation expressed in equation (16) can be used to approximate the shear thinning behavior for stresses greater than the yield level (f_0), and is shown in Figure 5.

Clay-Soil Behavior

The previous analysis outlines the establishment of soil properties in terms of rheologic behavior. It is thus possible to approximate the total stress-strain-time behavior of a clay in terms of its component behavior.

In volume change, the stress-strain behavior is expressed by the differential equation.

$$\frac{\partial^2 \epsilon_{ii}}{\partial t^2} + \frac{K_2}{\zeta_2}\frac{\partial \epsilon_{ii}}{\partial t} = \frac{1}{3K_1}\frac{\partial \sigma_{ii}}{\partial t^2} + \left[2\frac{\partial}{\partial t}(\frac{1}{3K_1})\right.$$

$$+ \frac{K_2}{3K_1\zeta_2} + \frac{1}{3\zeta_1} + \frac{1}{3\zeta_2}\left]\frac{\partial \sigma_{ii}}{\partial t} + \left[\frac{\partial^2}{\partial t^2}(\frac{1}{3K_1})\right.\right.$$

$$+ \frac{\partial}{\partial t}(\frac{K_2}{3K_1\zeta_2} + \frac{1}{3\zeta_1}) + \frac{K_2}{3K_1\zeta_1}\left]\sigma_{ii} \qquad (17)$$

The deviator stress-strain-time differential equation is:

$$s_{ij} = 2Ge_{ij} \qquad (s_{ij} < f_o) \qquad (18a)$$

$$\frac{\partial^2 e_{ij}}{\partial t^2} + \frac{G_2}{\eta_2}\frac{\partial e_{ij}}{\partial t} = \frac{1}{2G_1}\frac{\partial^2 s_{ij}}{\partial t^2} + \left[\frac{G_2}{2G_1\eta_2} + \frac{1}{2\eta_1} + \frac{1}{2\eta_2}\right]\frac{\partial s_{ij}}{\partial t}$$

$$+ \left[\frac{\partial}{\partial t}(\frac{1}{2\eta_1}) + \frac{G_2}{2G_1\eta_1}\right]s_{ij} \qquad (s_{ij} > f_o) \qquad (18b)$$

The initial conditions to equations (17) and (18) are dependent on the particular application and inherent soil initial stresses and strains.

GROUND SURFACE DEFORMATION

The deformation of the surface of the ground resulting from vehicular action is due to several factors. These are:

1. Speed of surface movement.

2. Imposed stresses as a function of vehicle weight.

Two problems of interest are those pertaining to a standing vehicle and a moving vehicle.

In order to achieve indicative solutions, a linear theory will be used.

Standing Vehicle

In volume change, the most critical problem of a standing vehicle is the long time deformation. For this case, the theory of consolidation is applicable.

The deviator behavior is a function of the vehicle load. For light loads where the imposed deviator stresses are less than (f_o) the behavior is elastic. For imposed deviator stresses greater than (f_o) visco-elastic behavior is requisite.

As a first example, a light vehicle is considered. The volume change is considered to act as a Kelvin material while the deviator behavior is elastic.

$$\sigma_{ii} = 3K\epsilon_{ii} + 3\zeta \frac{\partial \epsilon_{ii}}{\partial t} \tag{19a}$$

$$s_{ij} = 2G e_{ij} \qquad\qquad (s_{ij} < f_o) \tag{19b}$$

The visco-elastic operators are:

$$R = 1 \tag{20a}$$

$$S = 3K + 3\zeta \frac{\partial}{\partial t} \tag{20b}$$

$$M = 1 \tag{20c}$$

$$N = 2G \tag{20d}$$

The displacement time relation from equations (8) becomes:

$$\frac{\partial w}{\partial t} + \frac{3K+G}{3\zeta} w = \frac{3K+4G}{12G\zeta} P \tag{21a}$$

$$w(o) = w_O \tag{21b}$$

Equations (21) are the equations of a Kelvin model in which the soil is pre-deformed by a deformation (w_O)

$$w(t) = \frac{3K+4G}{4G(3K+G)} P + \left[w_o - \frac{3K+4G}{4G(3K+G)} P \right] e^{-t/T} \tag{22a}$$

$$T = \frac{3\zeta}{3K+G} \tag{22b}$$

The results presented in equations (22) indicate several features of the time-displacement relation. In the first place, the time retardation of elastic displacement is greater than would be expected by pure consolidation. The additional delay is due to the shear deformation.

Secondly, the previous load history plays an important factor
in the total deformation. At a given vehicle stand, if the
elastic deformation is less than the previous deformation
(w_o), the subsequent displacement will be a swell at the
ground. If the previous deformation (w_o) can be matched to
the elastic deformation, due the vehicle load, this vehicle
can stand indefinitely without any sinkage whatsoever. For
a given residual settlement and vehicle load, the zero defor-
mation condition can be established by adjusting the contact
area and geometry of the track or standing platform.

For some soft soils, it is not possible to keep the
deviator stress below (f_o). In these cases, the analysis
must be based on a time-dependent deviator stress-strain
relation. As an extreme example, the deviator behavior will
be considered as viscous.

$$\sigma_{ii} = 3K\epsilon_{ii} + 3\zeta \frac{\partial \epsilon_{ii}}{\partial t} \tag{23a}$$

$$s_{ij} = 2\eta^* \frac{\partial e_{ij}}{\partial t} \qquad (s_{ij} > f_o) \tag{23b}$$

The visco-elastic operators are:

$$R = 1 \tag{24a}$$

$$S = 3K + 3\zeta \frac{\partial}{\partial t} \tag{24b}$$

$$M = 1 \tag{24c}$$

$$N = 2\eta^* \frac{\partial}{\partial t} \tag{24d}$$

The displacement relation from equations (8) becomes:

$$\frac{\partial^2 w}{\partial t^2} + \frac{3K}{3\zeta+\eta^*}\frac{\partial w}{\partial t} = \frac{3K}{4\eta^*(3\zeta+\eta^*)}P \qquad (25a)$$

$$w(0) = 0 \qquad (25b)$$

The visco-elastic law is derived from a physical relationship in integral form. In order to convert to differential form, a second initial condition was built into the deformation relationship. Thus, equation (25a) is a second order equation. One method of converting back to the integral relation consists of finding a visco-elastic model which is represented by equations(25). Since the volume change and deviator coupling is essentially a series coupling, the load-deformation model is as shown in Figure 6. The load-deformation relation is:

$$w(t) = \frac{1}{4\eta^*}\int_0^t P\,dt + \frac{3}{4(3\zeta+\eta^*)}e^{-t/T}\int_0^t Pe^{t/T}dt \qquad (26a)$$

$$T = \frac{3\zeta+\eta^*}{3K} \qquad (26b)$$

For a given vehicle load (P) the deformation-time relation is:

$$w(t) = \frac{P}{4\eta^*}t + \frac{P}{4K}(1 - e^{-t/T}) \qquad (27)$$

As shown in Figure 7, the deformation increases with time, approaching a linear relationship.

From a design point of view, this relation indicates that for a given set of soil parameters, a given load and a given geometry of loaded area, the critical settlement with time can be established. Thus, the time of parking of a vehicle can be computed for field use.

The viscosity (η^*) is actually an apparent viscosity as indicated in Figure 4.

Moving Vehicle

The deformation-time analysis for a moving vehicle must include considerations of vehicle speed in addition to considerations of the soil properties and imposed stresses.

A high speed light vehicle will not develop any time-dependent volumetric stresses and will be in the range of elastic deviator stresses. The stress-strain relations for this type of activity are:

$$\sigma_{ii} = 3K_1 \epsilon_{ii} \tag{28a}$$

$$s_{ij} = 2G e_{ij} \tag{28b}$$

The displacements will be instantaneous and elastic in direct proportion to the time of loading.

$$w(t) = \frac{3K_1 + 4G}{4G(3K_1 + G)} P(t) \tag{29}$$

The same light vehicle for a relatively slow traverse of the ground will permit time dependent deformations in accordance with both the soil properties and the vehicle movement program.

$$\frac{\partial^2 \sigma_{ii}}{\partial t^2} + \left[\frac{K_2}{\zeta_2} + \frac{K_1}{\zeta_1} + \frac{K_1}{\zeta_2}\right]\frac{\partial \sigma_{ii}}{\partial t} + \frac{K_2}{\zeta_1}\sigma_{ii}$$

$$= 3K_1 \frac{\partial^2 \epsilon_{ii}}{\partial t^2} + \frac{3K_1 K_2}{\zeta_2}\frac{\partial \epsilon_{ii}}{\partial t} \tag{30a}$$

$$s_{ij} = 2G\, e_{ij} \tag{30b}$$

The visco-elastic operators are:

$$R = \frac{\partial^2}{\partial t^2} + \left[\frac{K_2}{\zeta_2} + \frac{K_1}{\zeta_1} + \frac{K_1}{\zeta_2}\right]\frac{\partial}{\partial t} + \frac{K_2}{\zeta_1} \tag{31a}$$

$$S = 3K_1 \frac{\partial^2}{\partial t^2} + \frac{3K_1 K_2}{\zeta_2}\frac{\partial}{\partial t} \tag{31b}$$

$$M = 1 \tag{31c}$$

$$N = 2G \tag{31d}$$

The basic load deformation equation is:

$$(3K_1 + G)\frac{\partial^2 w}{\partial t} + \left[\frac{3K_1 K_2}{\zeta_2} + \frac{GK_2}{\zeta_2} + \frac{GK_1}{\zeta_1} + \frac{GK_1}{\zeta_2}\right]\frac{\partial w}{\partial t} + \frac{2GK_2}{\zeta_1}w$$

$$= \frac{3K_1 + 4G}{4G}\frac{\partial^2 P}{\partial t^2} + \left[\frac{3K_1 K_2}{4G\zeta_2} + \frac{K_2}{\zeta_2} + \frac{K_1}{\zeta_1} + \frac{K_1}{\zeta_2}\right]\frac{\partial P}{\partial t} + \frac{K_2}{\zeta_1}P \tag{32a}$$

$$w(0) = 0 \qquad\qquad (32b)$$

The differential equations (32a) can be represented
by a five element visco-elastic model as shown in Figure 8.
For this particular problem, the relation between the model
parameters and the moduli and viscosities of the soil will
not be developed since they do not help the analysis.

The load-deformation-time relation for the five
element model is:

$$w = \frac{P}{A_1} + \frac{1}{B_2}e^{-t/T_2}\int_0^t P e^{t/T_2}dt + \frac{1}{B_3}e^{-t/T_3}\int_0^t P e^{t/T_3}dt \qquad (33a)$$

$$T_2 = B_2/A_2 \qquad\qquad (33b)$$

$$T_3 = B_3/A_3 \qquad\qquad (33c)$$

A single passage of the load in time (t) results in
a load-deformation relation as follows:

$$w = \frac{P}{A_1} + \frac{P}{A_2}\left[1 - e^{-t/T_2}\right] + \frac{P}{A_3}\left[1 - e^{-t/T_3}\right] \qquad (34)$$

Thus, the vehicle passage will result in an instan-
taneous elastic deformation followed by additional viscous
deformations. The longer the time of passage (the slower
the vehicle) the greater the deformation. From the point of
mobility, it can be seen that a vehicle with a limiting

deformation characteristic can be designed for a particular soil in terms of its time of passage.

The principles of design of a vehicle train can also be established by examining the five element model. The degree of relaxation of deformation is an exponential function of the speed and spacing between vehicles. For a given vehicle, spacing and speed, the deformation will increase with length of train. Using a limiting deformation as a design criteria, the number of vehicles can be increased by increasing their spacing.

Another consideration is the relatively high speed movement of a heavy vehicle over a soft soil. The volumetric stress-strain relation will be elastic while the deviator stress-strain relation can be considered to be viscous with an apparent viscosity (η^{*})

$$\sigma_{ii} = 3K_1\epsilon_{ii} \qquad (35a)$$

$$s_{ij} = 2\eta^{*}\frac{\partial e_{ij}}{\partial t} \qquad (35b)$$

The operator relations are:

$$R = M = 1 \qquad (36a)$$

$$S = 3K_1 \qquad (36b)$$

$$V = 2\eta^{*}\frac{\partial}{\partial t} \qquad (36c)$$

The load deformation differential equation then becomes:

$$\frac{\partial^2 w}{\partial t^2} + \frac{3K_1}{\eta^*} \frac{\partial w}{\partial t} = \frac{1}{\eta^*} \frac{\partial P}{\partial t} + \frac{3K}{4(\eta^*)^2} P \tag{37a}$$

$$w(0) = 0 \tag{37b}$$

The model which produces equation (37a) is a three-element model similar to the one shown in Figure 6. The only difference is the value of the Kelvin dash-pot.

The deformation response of this model is:

$$w(t) = \frac{1}{4\eta^*} \int_0^t P\, dt + \frac{3}{4\eta^*} e^{-t/T} \int_0^t P\, e^{t/T}\, dt \tag{38a}$$

$$T = \eta^*/3K_1 \tag{38b}$$

For a single traverse in time (t) the deformation becomes:

$$w(t) = \frac{P}{4\eta^*} t + \frac{P}{4K_1} (1 - e^{-t/T}) \tag{39a}$$

$$\frac{\partial w}{\partial t}(0) = \frac{P}{\eta^*} \tag{39b}$$

The relation in equations (39) is the same as shown in Figure 7 with a different magnitude of the initial slope.

As the load increases, the viscosity will decrease and the apparent viscosity (η^*) will drop very swiftly. As a result, the limiting design deformation will be approached exponentially, with load and time of application. Thus, the.

mobility design must be balanced between load and speed. Higher loads will require greater speeds for a given deformation.

The effect of vehicle trains on the viscous portion of the deformation relation will be particularly pronounced. The first term of equation (39a) indicate that the purely viscous portion of the deformation will never achieve recovery. Thus, successive vehicles in a train will impose linear increases on the deformation. The relaxation of the Kelvin unit will aid in the reduction of the total deformation, but it may not be enough to offset the large viscous deformations. As indicated previously, the proper train design will depend on the magnitude of the soil parameters and the balancing of the load and time of traverse.

By continuing the previous analysis, a vehicle load-deformation-time equation can be established for any combination of soil parameters. Furthermore, similar mobility design criteria can be established.

CONCLUSIONS

The foregoing analysis has attempted to develop rational relations for the soil parameters of importance in vehicle mobility, and to use the relations in terms of mobility design.

The mobility design was based on an assumed criteria, that the stresses imposed on the soil never reached the soil shear strength. This criteria was established since the achievement of shear strength by a single vehicle passage will remould the soil structure and thus make conditions of subsequent passage considerably less favorable.

The mobility design is a limiting deformation. Such a criteria can be established in terms of the type and desired speed of the vehicle.

The soil properties that have been established are:

1. The volume change behavior consists of an initial elastic deformation followed by a viscous delay, on loading. Unloading results in a partial elastic recovery followed by partial viscous recovery. Repeated load-unload cycles will increase the net deformation up to a deformation limit. Beyond the deformation limit, the soil will behave in a completely elastic manner.

2. The deviator behavior on loading consists of elastic behavior up to a yield stress (f_0). Beyond (f_0) the soil behaves as a shear thinning material with decreasing viscosity until shear failure is reached.

An analysis of load-deformation-time response for various conditions of loading, and soil conditions indicated the following:

1. The deformation of light vehicles when parked followed a Kelvin visco-elastic relation. The deformation could be controlled to tolerable limits by altering the area and geometry of the loaded area. For soft soils this can be accomplished by using parking mats.

2. The deformation of heavy parked vehicles follows a Kelvin visco-elasticity augmented by a purely viscous deformation. In this case, deformation can only be controlled by restricting the time of parking. The maximum parking time is a function of the load and magnitude of the soil properties.

3. A light fast vehicle will impose only elastic deformations on the ground.

4. Heavy vehicles can be designed for a mobility criteria (maximum deformation), based on speed, load and magnitude of soil properties.

5. The design of vehicle trains depends on the balancing of the load and vehicle spacing to provide sufficient deformation recovery to satisfy the mobility criteria. Another design feature is the total deformation as a function of the number of vehicles in the train.

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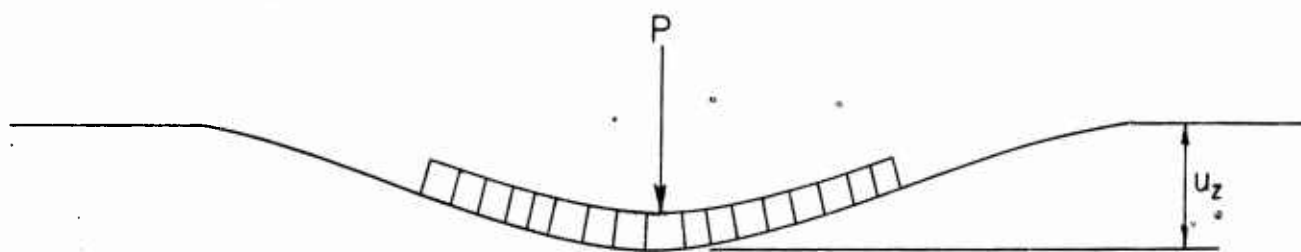
FIGURE 1

LOADED PLATE ON HALF SPACE

G_1 K_1

η_1 ζ_1

G_2 K_2 η_2 ζ_2

σ_0 τ ϵ_0 γ
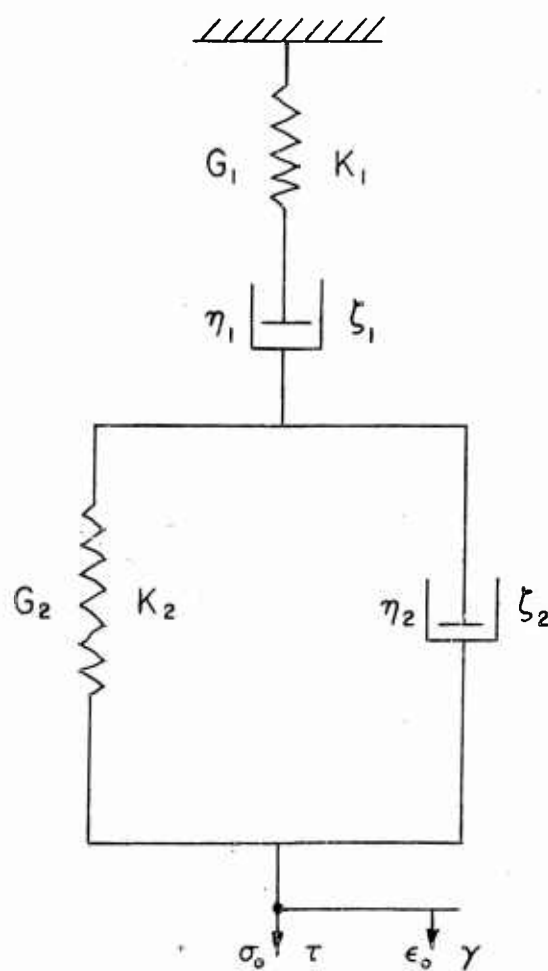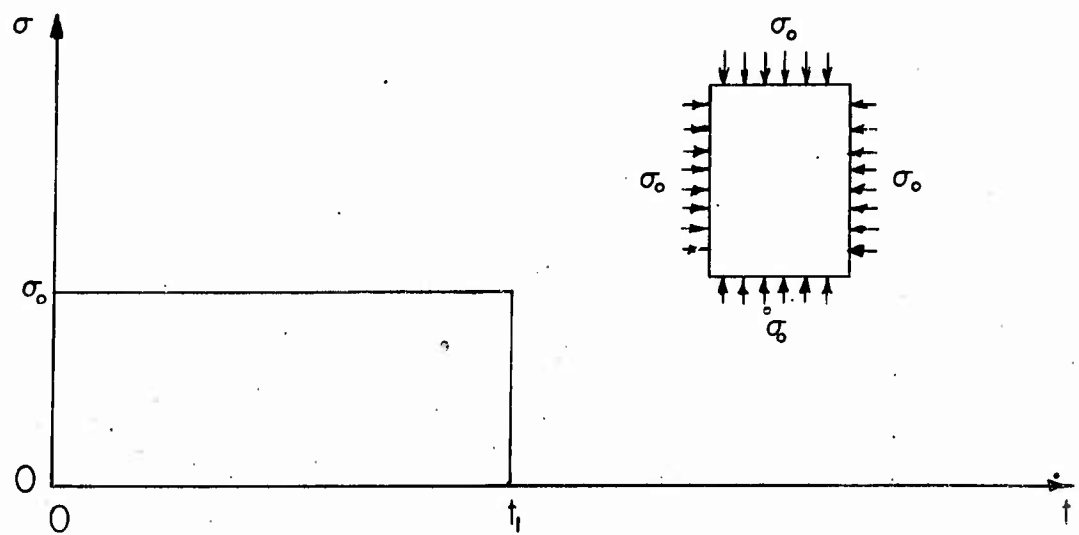
FIGURE 2

BURGERS MODEL
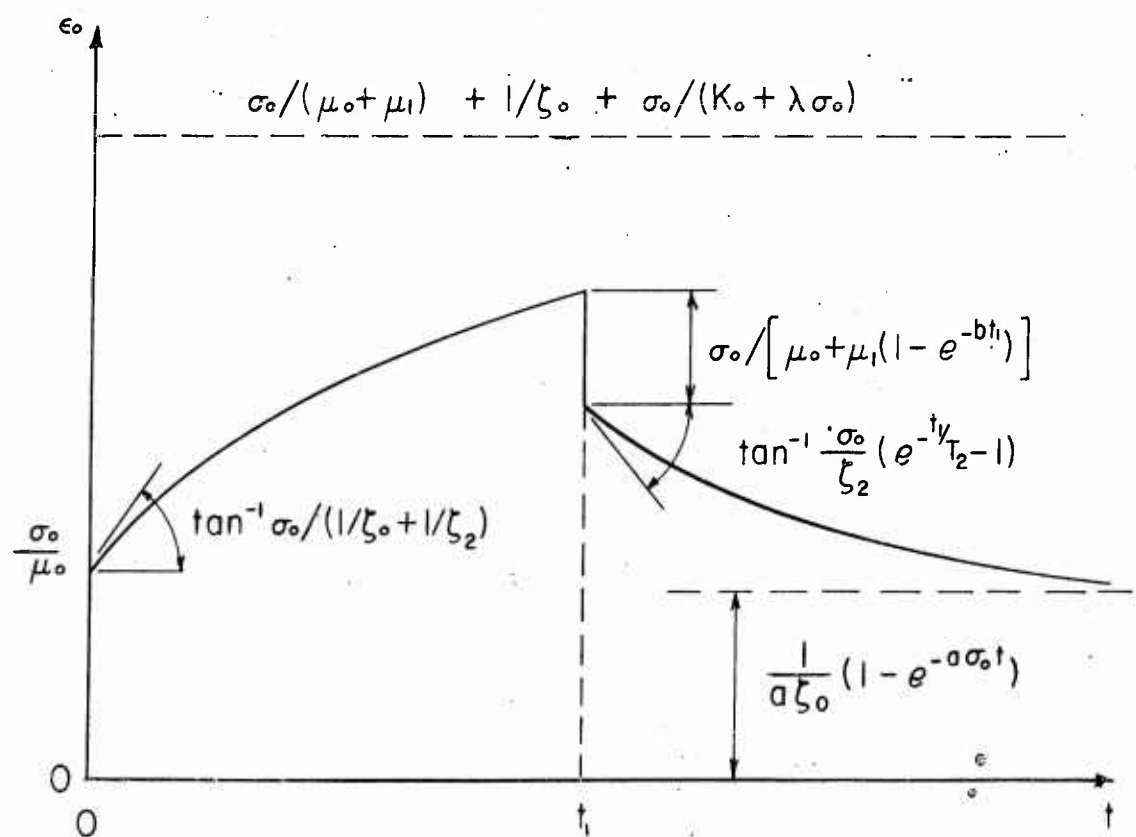
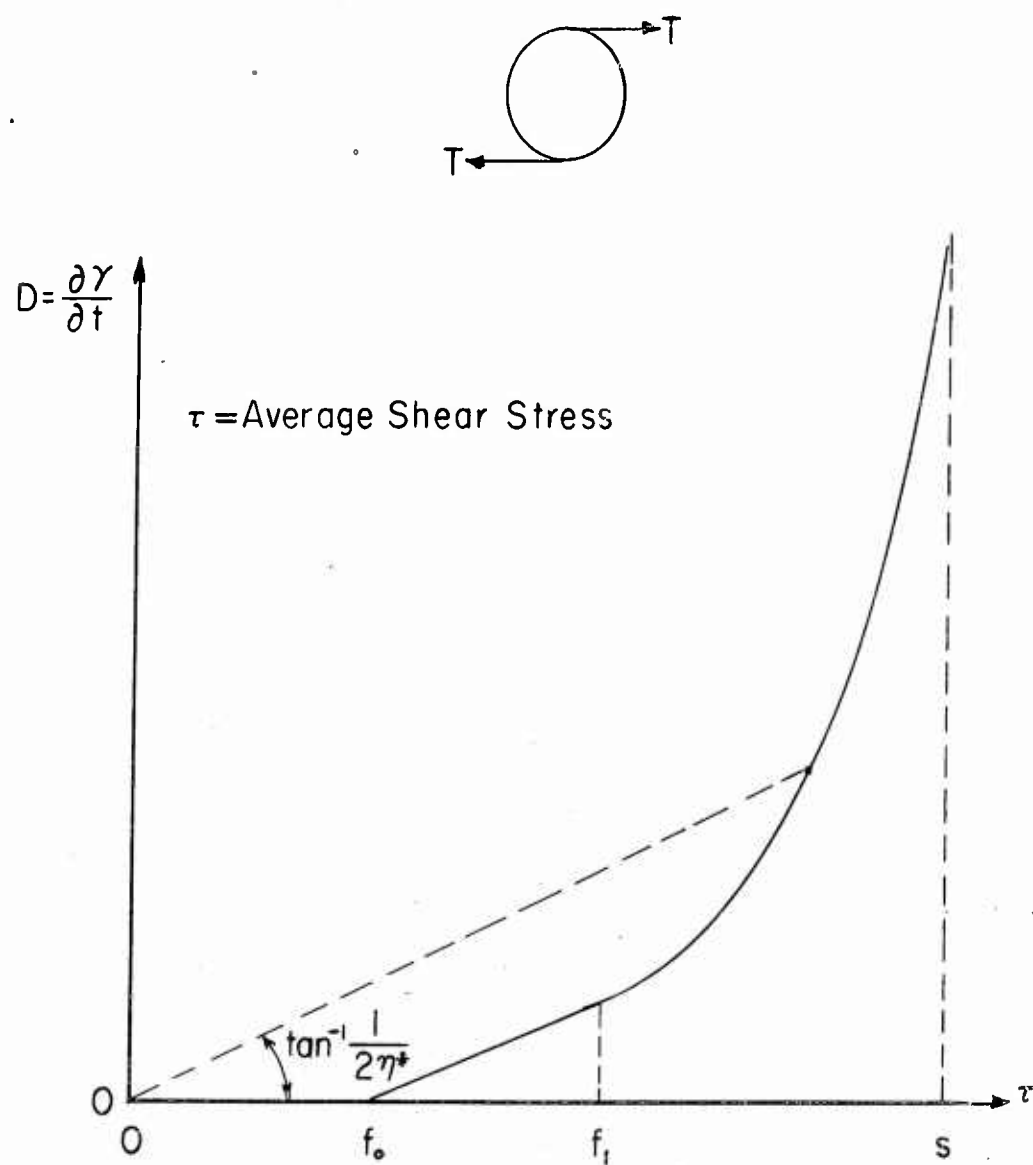FIGURE 3

VOLUME CHANGE CLAY BEHAVIOR
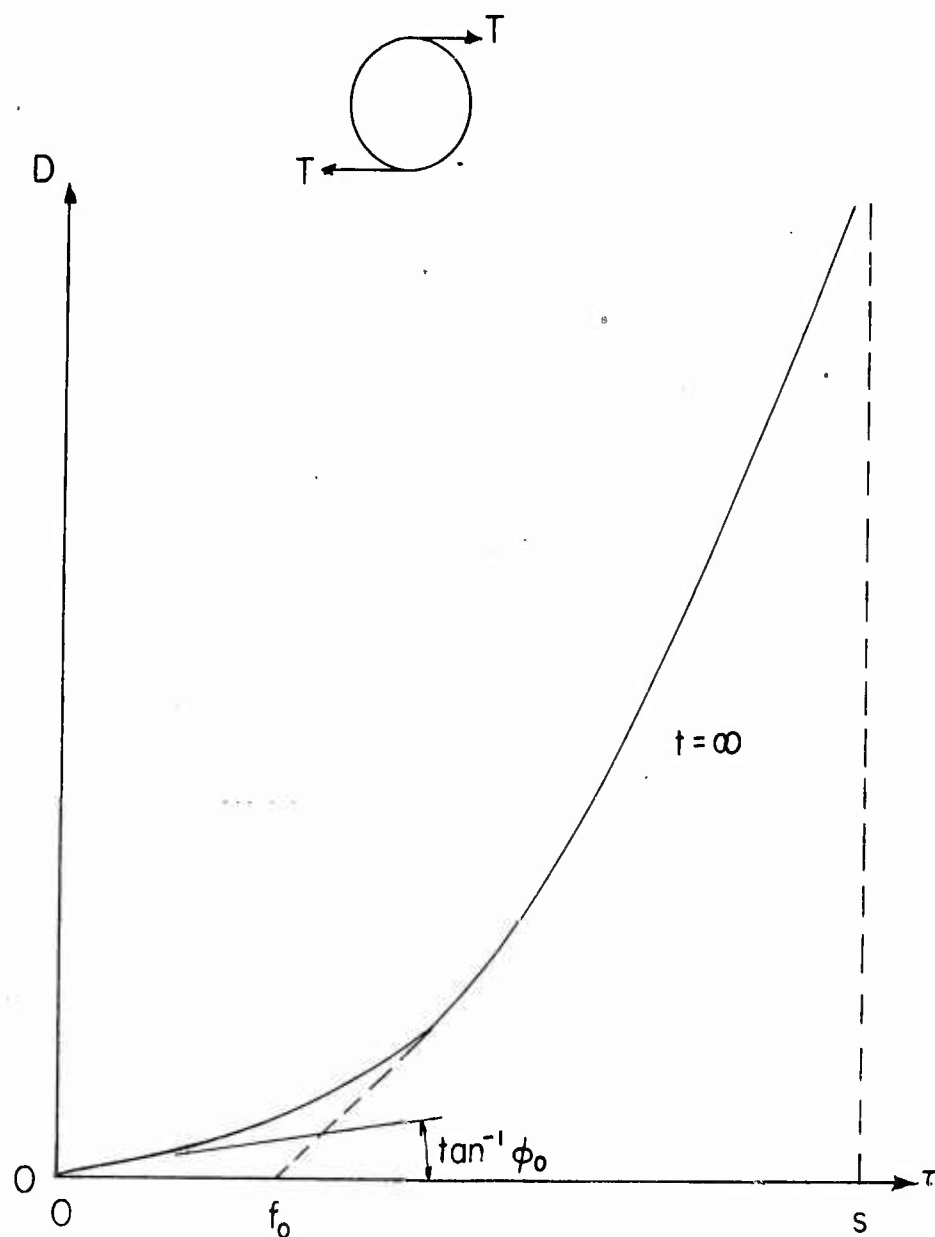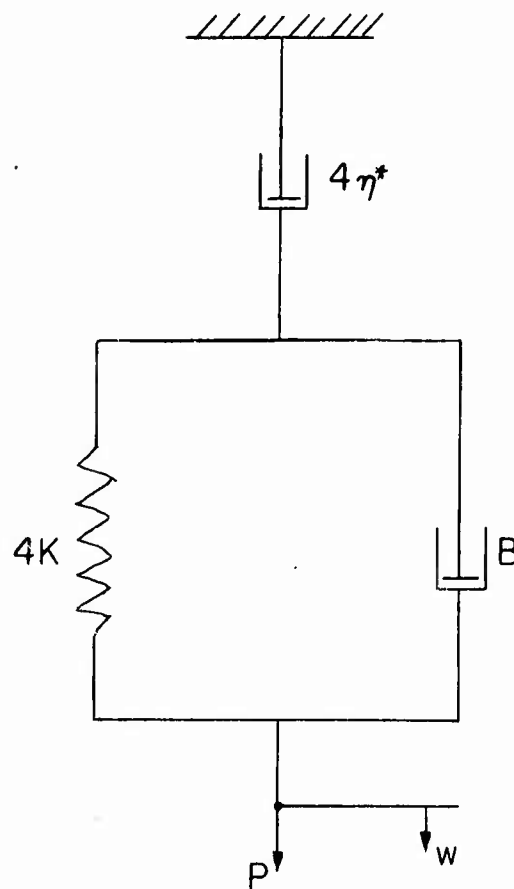
FIGURE 4

DEVIATOR BEHAVIOR OF CLAY

FIGURE 5
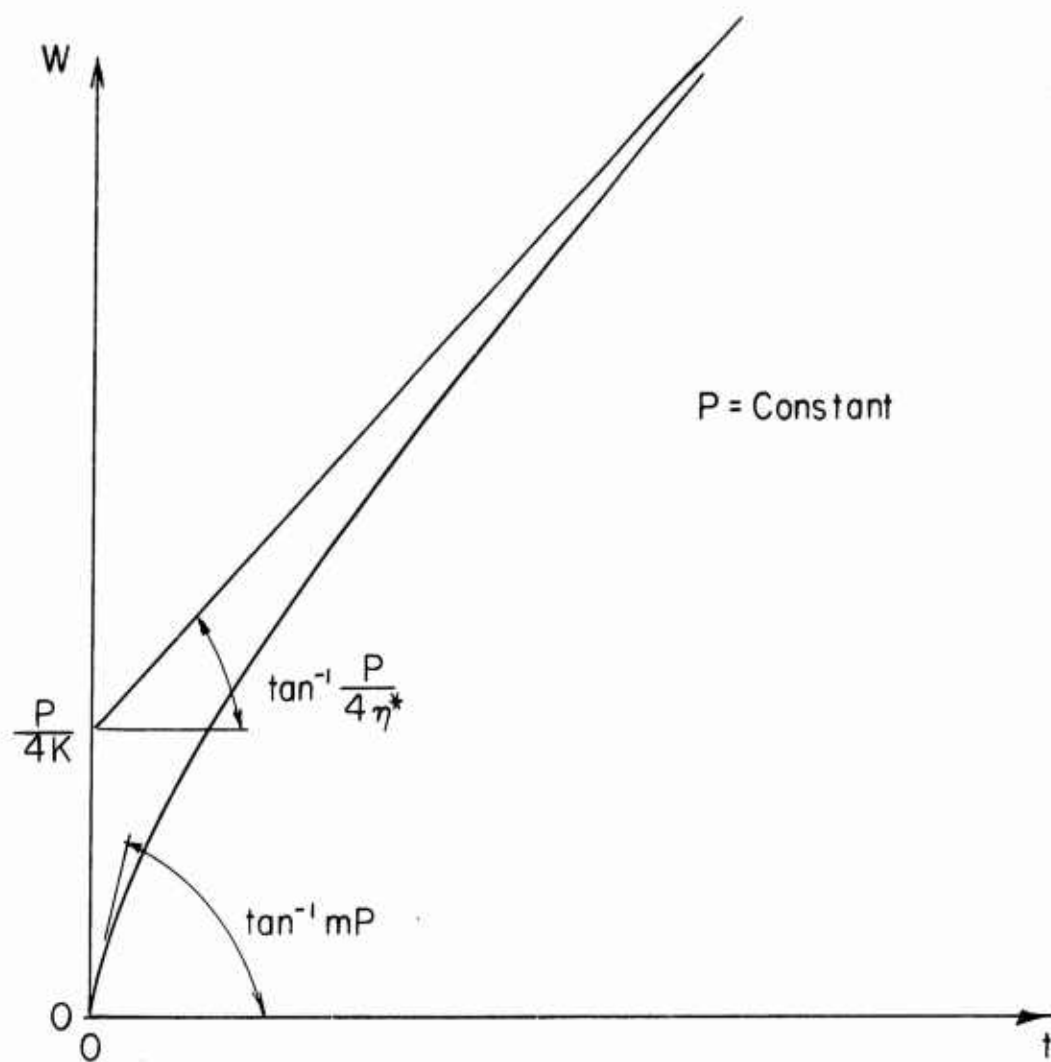
SHEAR BEHAVIOR OF CLAY

$(\tau > f_0)$

Consolidating-Viscous Soil: $B = \dfrac{4(3\zeta + \eta^*)}{3}$

Elastic-Viscous Soil: $B = \dfrac{4\eta^*}{3}$

FIGURE 6

LOAD-DEFORMATION MODEL

Consolidating-Viscous Soil: $m = \dfrac{3\zeta + 4\eta^{*}}{4\eta^{*}(3\zeta + \eta^{*})}$
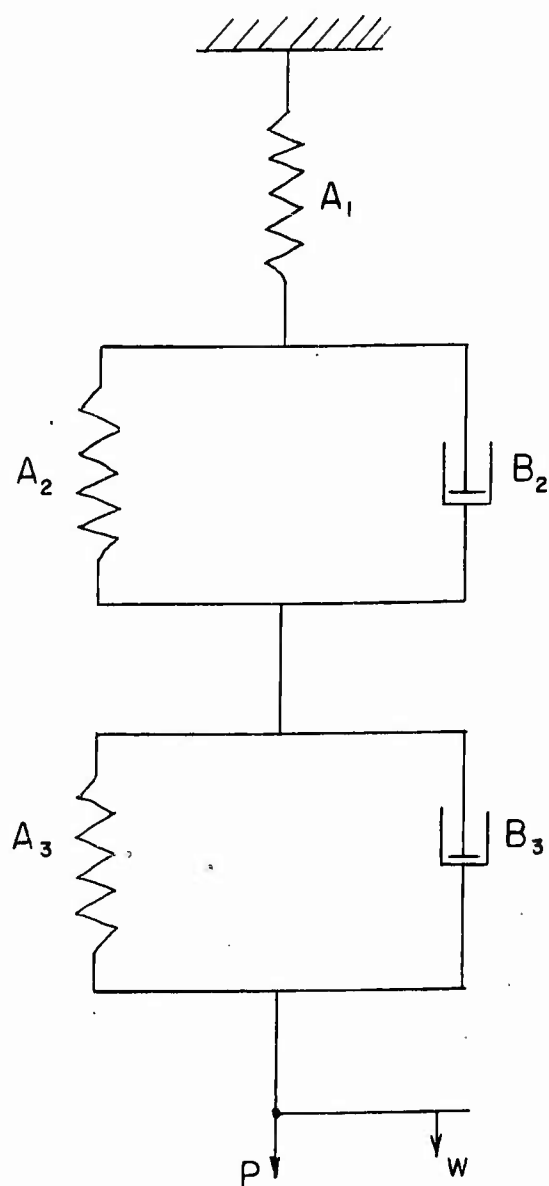
Elastic-Viscous: $m = 1/\eta^{*}$

FIGURE 7

DEFORMATION—TIME RELATION

FIGURE 8

FIVE-ELEMENT VISCO-ELASTIC MODEL